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$$(1-x^2) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 0.$$

Solving the differential equation, we have

$$y = \frac{c}{2} [x \sqrt{1-x^2} + \sin^{-1} x].$$

To verify that this gives the given series, we have by differentiating,

$$\frac{dy}{dx} = c \sqrt{1-x^2} = c \left(1 - \frac{1}{2} x^2 - \frac{1}{2! 2^2} x^4 - \frac{1.3}{3! 2^3} x^6 - \dots \right.$$

$$\left. - \frac{1.3.5 \dots (2n-1)}{(n+1)! 2^{n+1}} x^{2n+2} - \dots \right).$$

$$\text{Integrating, } y = c \left(x - \frac{1}{2.3} x^3 - \frac{1}{2! 2^2 .5} x^5 - \frac{1.3}{3! 2^3 .7} x^7 - \dots \right.$$

$$\left. - \frac{1.3.5 \dots (2n-1)}{(n+1)! 2^{n+1} (2n+3)} x^{2n+3} - \dots \right).$$

Putting $c = -2$, we have

$$\lim_{x \rightarrow 1} [- (x \sqrt{1-x^2} + \sin^{-1} x)] = -\frac{\pi}{2} = -2 + \frac{1}{3} + \frac{1}{2! 2.5} + \frac{1.3}{3! 2^3 .7} + \dots$$

Also solved by G. B. M. Zerr, who found for the limit of the sum of n terms of the series as $n \rightarrow \infty$, $\frac{\pi}{2} - \frac{1}{2}\pi$.

GEOMETRY.

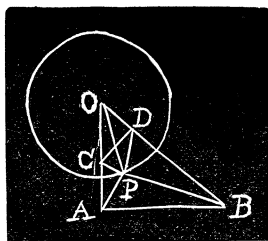
349. Proposed by J. A. CAPARO, Notre Dame University, Notre Dame, Indiana.

Given the radius of a circular smooth cylinder and its position with respect to a source of light and the eye. Find a geometrical construction to determine the line of brilliancy.

Solution by C. N. SCHMALL, New York City.

By a simple examination of the data* it is clear that the problem reduces to the following: Given a circle and two points A , B , outside (in same plane), to find a point P on the circumference, such that AP and

*See *Encyclopedia Britannica*, Vol. XIV, page 589.



BP make equal angles with the radius drawn to P .

This is known as Alhazen's Problem and does not admit of a solution by ruler and compasses only. However, by the use of an hyperbola an approximate solution can be effected.

Construction. Let O be the center of the circle, a its radius. Take C and D so that $AO \cdot OC = a^2 = BO \cdot OD$.

Now, the locus of the vertices of the triangles whose base is CD and whose base angles have a constant difference ($\angle OCD - \angle ODC$) is well known to be a hyperbola. This will cut the circle in four points, of which let P be one. This is the point required.

Proof. $\angle CDP - \angle DCP = \angle OCD - \angle ODC$ (by construction). Transposing and adding,

$$\therefore \angle OCP = \angle ODP \dots (1).$$

Also in the triangles AOP , POC , we have $AO:OP = OP:OC$ (by construction).

$$\therefore \angle APO = \angle OCP. \text{ Similarly, } \angle BPO = \angle ODP \text{ by (1).}$$

$$\therefore \angle APO = \angle BPO, \text{ and } \therefore \angle APR = \angle BPR. \quad \text{Q. E. F. Q. E. D.}$$

350. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Given the quadrilateral $AB=a=225$, $BC=b=153$, $CD=c=207$, $DA=d=135$, $AC=e=240$. Find the side of the square inscribed in this quadrilateral having a corner in each side.

Solution by the PROPOSER.

Let $ABCD$ be the given quadrilateral, $EFGH$ the inscribed square, $RPQS$ the circumscribed rectangle having its sides parallel to the sides of the square. Draw AIJ , BUT , CNL , DVM perpendicular, respectively, to EF and HG , FG and HE , HG and EF , HE and GF .

Let $AB=a$, $BC=b$, $CD=c$, $DA=d$, $AC=e$, $BD=f$, O the intersection of AC , BD , $\angle COB = \beta$, $\angle ACL = \theta = \angle CAJ$, $\angle BDM = \phi = \angle DBT$, $\angle BAC = \delta$, $\angle DAC = \gamma$, $\angle BCA = \rho$, $\angle DCA = \mu$, area $ABCD = \Delta$.

Then $\phi = \frac{1}{2}\pi - (\beta - \theta)$, $RB = a \cos(\delta - \theta)$, $RA = a \sin(\delta - \theta)$, $DS = d \cos(\gamma + \theta)$, $AS = d \sin(\gamma + \theta)$, $DQ = c \cos(\mu - \theta)$, $CQ = c \sin(\mu - \theta)$, $BP = b \cos(\rho + \theta)$, $PC = b \sin(\rho + \theta)$, $AI + x + CN = AJ + CN = e \cos \theta$, $BU + x + DV = BT + DV = f \cos \phi$.

$$\therefore BT + DV = f \sin(\beta - \theta).$$

$$x^2 + \frac{1}{2}x(AI + BU + CN + DV) = \Delta = \frac{1}{2}x(AI + x + CN) + \frac{1}{2}x(BU + x + DV).$$

$$\therefore x[ecos \theta + fsin(\beta - \theta)] = 2\Delta \dots (1).$$

